Membership Conditional Term Rewriting Systems

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Summary

We propose a new type of conditional term rewriting system: the membership-conditional term rewriting system, in which, each rewriting rule can have membership conditions which restrict the substitution values for the variables occurring in the rule. For example, the rule $f(x, x, y) \triangleright g(x, y)$ *if* $x \in T'$ yields the reduction $f(M, M, N) \to g(M, N)$ only when M is in the term set T'. We study the confluence of membership-conditional term rewriting systems that are *nonterminating* and *nonlinear*. It is shown that a restricted *nonlinear* term rewriting system in which membership conditions satisfy the closure and termination properties is confluent if the system is nonoverlapping.

1. Introduction

Many term rewriting systems and their modifications are considered in logic, automated theorem proving, and programming language [2, 3, 4, 6, 7, 9, 10]. A fundamental property of term rewriting systems is the confluence property. A few sufficient criteria for the confluence are well known [2, 3, 4, 5, 9, 10]. However, if a term rewriting system is nonterminating and nonlinear, we know few criteria for the confluence of the system [8, 11].

In this paper, we study the confluence of membership-conditional term rewriting systems that are nonterminating and nonlinear. In a membership-conditional term rewriting system, the rewriting rule can have membership conditions.

We explain this concept with an example. We first consider a classical term rewriting system R that is nonterminating and nonlinear:

$$R \quad \begin{cases} f(x, x) \triangleright 0 \\ f(g(x), x) \triangleright 1 \\ 2 \triangleright g(2) \end{cases}$$

The diagram in Figure 1 illustrates that R is not confluent:

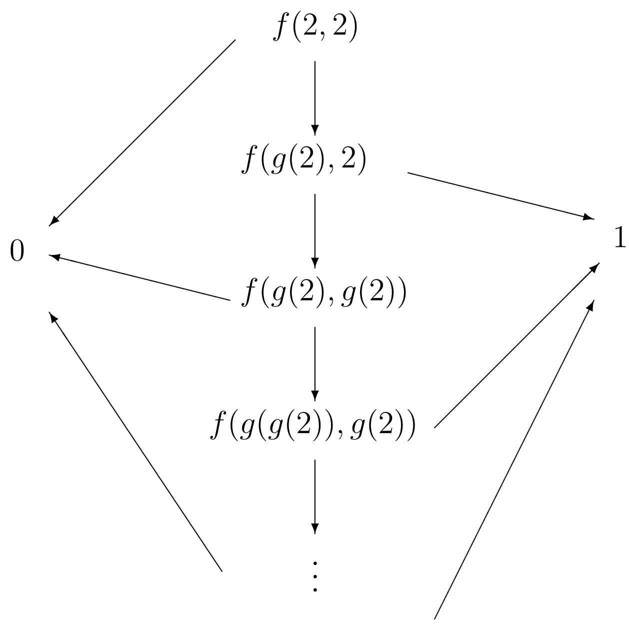

Figure 1. Reductions starting with $f(2,2)$.

Now, let T' be the set of terms containing no constant symbol 2. By adding the membership condition $x \in T'$ to the first and second rules in R, we obtain the membership-conditional term rewriting system R':

$$R' \quad \begin{cases} f(x,x) \rhd 0 \textit{ if } x \in T' \\ f(g(x),x) \rhd 1 \textit{ if } x \in T' \\ 2 \rhd g(2) \end{cases}$$

The membership condition $x \in T'$ restricts the substitution values for variable x; for example, the first rule $f(x,x) \rhd 0 \textit{ if } x \in T'$ defines the reduction $f(M,M) \to 0$ only when $M \in T'$. Then, we can prove that R' is confluent (see Example 5.2 in Section 5), though it is nonterminating and nonlinear. Thus, by adding appropriate membership conditions, nonlinear systems can easily have the confluence property.

Our idea of membership-conditional rewriting was inspired by Church's δ-rule in λ-calculus [1, 8]:

$$\delta_C \quad \begin{cases} \delta MM \rhd \mathbf{T} \;\; \textit{if } M \textit{ is a closed normal form} \\ \delta MN \rhd \mathbf{F} \;\; \textit{if } M, N \textit{ are closed normal forms and } M \not\equiv N. \end{cases}$$

It is well known that λ-calculus with δ_C is confluent [1, 8]. However, if λ-calculus has Hindley's δ-rule

$$\delta_H \quad \{ \; \delta MM \rhd M$$

or Staples's δ-rule

$$\delta_S \quad \{ \; \delta MM \rhd \epsilon$$

instead of δ_C, then it is not confluent [1, 8]. Thus, the membership conditions in δ_C (i.e., M, N *must be in the set of closed normal forms*) play an important role for the confluence of λ-calculus with nonlinear rules.

We will extend the idea of membership-conditional rewriting offered in Church's δ-rule to nonlinear term rewriting systems. Section 2 and Section 3 introduce preliminary concepts of reduction systems and of term rewriting systems respectively. In the next section, we present the concept of membership-conditional term rewriting systems. In Section 5, we discuss the sufficient criteria for the confluence of membership-conditional term rewriting systems that are nonterminating and nonlinear. We show that a restricted nonlinear system in which the membership conditions satisfy the closure and termination properties is confluent if the system is nonoverlapping.

2. Reduction Systems

We explain notions of reduction systems and give definitions for the following sections. Since these reduction systems have only an abstract structure, they are called abstract reduction systems [3, 8] .

A reduction system is a structure $R = \langle A, \rightarrow \rangle$ consisting of some object set A and some binary relation $\rightarrow$ on A (i.e., $\rightarrow \subseteq A \times A$), called a reduction relation. A reduction (starting with x_0) in R is a finite or infinite sequence $x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow \cdots$. The identity of elements of A (or syntactical equality) is denoted by $\equiv$. $\stackrel{*}{\rightarrow}$ is the transitive reflexive closure of $\rightarrow$ and $=$ is the equivalence relation generated by $\rightarrow$ (i.e., the transitive reflexive symmetric closure of $\rightarrow$). If $x \in A$ is minimal with respect to $\rightarrow$, i.e., $\neg \exists y \in A[x \rightarrow y]$, then we say that x is a normal form, or $\rightarrow$ normal form; let NF be the set of normal forms. If $x \stackrel{*}{\rightarrow} y$ and $y \in NF$ then we say x has a normal form y and y is a normal form of x. $x \downarrow$ indicates a normal form of x.

Definition. $R = \langle A, \rightarrow \rangle$ is terminating (or $\rightarrow$ is terminating), iff every reduction in R terminates, i.e., there is no infinite sequence $x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow \cdots$.

Definition. $R = \langle A, \rightarrow \rangle$ is confluent (or $\rightarrow$ is confluent), iff
$\forall x, y, z \in A[x \stackrel{*}{\rightarrow} y \wedge x \stackrel{*}{\rightarrow} z \Rightarrow \exists w \in A, y \stackrel{*}{\rightarrow} w \wedge z \stackrel{*}{\rightarrow} w]$.

We express this property with the diagram in Figure 2. In this sort of diagram, dashed arrows denote (existential) reductions depending on the (universal) reductions shown by full arrows.

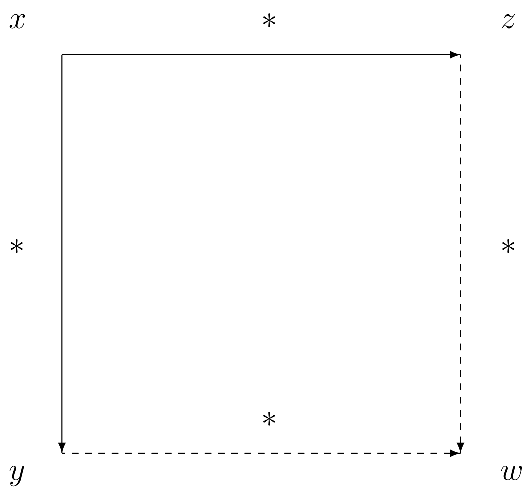

Figure 2. The confluence property.

The following propositions are well known [1, 3, 8] .

Proposition 2.1. Let R is confluent, then,

(1) $\forall x, y \in A[x = y \Rightarrow \exists w \in A, x \xrightarrow{*} w \wedge y \xrightarrow{*} w]$,

(2) $\forall x, y \in NF[x = y \Rightarrow x \equiv y]$,

(3) $\forall x \in A \forall y \in NF[x = y \Rightarrow x \xrightarrow{*} y]$.

3. Term Rewriting Systems

Term rewriting systems are reduction systems having a term set as an object set A. Assuming that the reader is familiar with the basic concepts concerning term rewriting systems, we briefly summarize the important notions below [3, 4].

Let F be an enumerable set of function symbols denoted by $f, g, h, \cdots$, and let V be an enumerable set of variable symbols denoted by $x, y, z, \cdots$ where $F \cap V = \phi$. By $T(F, V)$, we denote the set of terms constructed from F and V. If V is empty, $T(F, V)$, denoted as $T(F)$, is the set of ground terms. A term set is sometimes denoted by T.

A substitution θ is a mapping from a term set $T(F, V)$ to $T(F, V)$ such that for term M, $\theta(M)$ is completely determined by its values on the variable symbols occurring in M. Following common usage, we write this as $M\theta$ instead of $\theta(M)$.

Consider an extra constant $\Box$ called a hole and the set $T(F \cup \{\Box\}, V)$. Then $C \in T(F \cup \{\Box\}, V)$ is called a context on F. We use the notation $C[\ ,\dots,\ ]$ for the context containing n holes ($n \geq 0$), and if $N_1, \dots, N_n \in T(F, V)$, then $C[N_1, \dots, N_n]$ denotes the result of placing $N_1, \dots, N_n$ in the holes of $C[\ ,\dots,\ ]$ from left to right. In particular, $C[\ ]$ denotes a context containing precisely one hole. N is called a subterm of $M \equiv C[N]$.

A rewriting rule on T is a pair $\langle M_l, M_r \rangle$ of terms in T such that $M_l \notin V$ and any variable in M_r also occurs in M_l. The notation $\triangleright$ denotes a set of rewriting rules on T and we write $M_l \triangleright M_r$ for $\langle M_l, M_r \rangle \in \triangleright$. A $\rightarrow$redex, or redex, is a term $M_l\theta$, where $M_l \triangleright M_r$. The set $\triangleright$ of rewriting rules on T defines a reduction relation $\rightarrow$ on T as follows:

$$M \to N \text{ iff } M \equiv C[M_l\theta],\ N \equiv C[M_r\theta], \text{ and } M_l \rhd M_r \quad \text{for some } M_l, M_r, C[\ ], \text{ and } \theta.$$

When we want to specify the redex occurrence $\Delta \equiv M_l\theta$ of M in this reduction, write $M \stackrel{\Delta}{\to} N$.

Definition. A term rewriting system R on T is a reduction system $R = \langle T, \to \rangle$ such that the reduction relation $\to$ is defined by a set $\rhd$ of rewriting rules on T. If R has $M_l \rhd M_r$, then we write $M_l \rhd M_r \in R$.

If every variable in term M occurs only once, then M is called linear. We say that R is left-linear (or linear) iff for any $M_l \rhd M_r \in R$, M_l is linear. R is called nonlinear if R is not left-linear.

Let $M \rhd N$ and $P \rhd Q$ be two rules in R. We assume that we have renamed the variables appropriately, so that M and P share no variables. Assume $S \notin V$ is a subterm occurrence in M, i.e., $M \equiv C[S]$, such that S and P are unifiable, i.e., $S\theta \equiv P\theta$, with a minimal unifier θ [3, 9] . Since $M\theta \equiv C[S]\theta \equiv C\theta[P\theta]$, two reductions starting with $M\theta$, i.e., $M\theta \to C\theta[Q\theta] \equiv C[Q]\theta$ and $M\theta \to N\theta$, can be obtained by using $P \rhd Q$ and $M \rhd N$. Then we say that $P \rhd Q$ and $M \rhd N$ are overlapping, and that the pair $\langle C[Q]\theta, N\theta \rangle$ of terms is critical in R [3, 4] . We may choose $M \rhd N$ and $P \rhd Q$ to be the same rule, but in this case we shall not consider the case $S \equiv M$, which gives the trivial pair $\langle N, N \rangle$. If R has no critical pair, then we say that R is nonoverlapping [3, 4, 9, 11] .

The following sufficient conditions for the confluence of R are well known [3, 4, 9, 10] .

Proposition 3.1. Let R be terminating, and let P and Q have the same normal form for any critical pair $\langle P, Q \rangle$ in R. Then R is confluent.

Proposition 3.2. Let R be left-linear and nonoverlapping. Then R is confluent.

For more discussions concerning the confluence of term rewriting systems having overlapping or nonlinear rules, see [3, 8, 11] .

4. Membership-Conditional Rewriting

In this section, we propose membership-conditional term rewriting systems. A membership-conditional term rewriting system R on T is a term rewriting system on T in which the rewriting rule $M_l \rhd M_r$ can have the membership conditions $x \in T', y \in T'', \cdots, z \in T'''$. Here, $T', T'', \cdots, T'''$ are any subsets of T.

The membership-conditional rewriting rule is denoted by

$$M_l \rhd M_r \ \textit{if}\ x \in T', y \in T'' \cdots, z \in T'''.$$

The conditions $x \in T', y \in T'' \cdots, z \in T'''$ restrict the substitution's values on the variables $x, y, \cdots, z$ occurring in the rule $M_l \rhd M_r$. Thus, the rule $M_l \rhd M_r$ *if* $x \in T', y \in T'' \cdots, z \in T'''$ defines the reduction $M \to N$ only when $M \equiv C[M_l\theta]$, $N \equiv C[M_r\theta]$ for some $C[\ ]$ and some θ such that $x\theta \in T', y\theta \in T'', \cdots, z\theta \in T'''$.

Example 4.1. Let $F = \{+, d, s, 0\}$ and $F' = \{+, s, 0\}$. Consider the membership-conditional term rewriting system R on $T(F, V)$ which computes the addition and the double function $d(n) = n + n$ on the set $\mathbf{N}$ of natural numbers represented by $0, s(0), s(s(0)), \ldots$:

$$R \quad \begin{cases} x + 0 \triangleright x \\ x + s(y) \triangleright s(x + y) \\ d(x) \triangleright x + x \ \textit{if}\ x \in T(F') \end{cases}$$

Then we have the following reduction:

$$d(d(0)) \to d(0 + 0) \to (0 + 0) + (0 + 0) \overset{*}{\to} 0.$$

Note that $d(d(0))$ cannot directly contract into $d(0) + d(0)$ with the third rule in R since $d(0) \notin T(F')$. □

Example 4.2. Let $F = \{-, s, 0\}$. Consider the membership-conditional term rewriting system R on $T(F, V)$ computing the subtraction on the set $\mathbf{N}$:

$$R \quad \begin{cases} x - 0 \triangleright x \ \textit{if}\ x \in NF \\ s(x) - s(y) \triangleright x - y \ \textit{if}\ x, y \in NF \\ x - x \triangleright 0 \ \textit{if}\ x \in NF \end{cases}$$

Then, R contracts only the innermost redex occurrences in a term since the membership conditions prohibit to contract the other redex occurrences. Thus, by using the membership conditions we can explicitly provide the innermost reduction strategy for term rewriting systems. □

Note that we allow any (not necessarily decidable) membership condition $x \in T'$. However, if a membership condition is undecidable, the membership-conditional system R might not be well-defined (i.e., the reduction relation $\to$ of R cannot be defined). For example, a rewriting rule in which a membership condition restricts the application of itself leads us to the following paradoxical system R:

$$R \quad \{ \ f(x) \triangleright 0 \ \textit{if}\ x \in \{M | f(M) \in NF\}$$

Then, we can show that $f(0)$ is a normal form iff $f(0)$ is not a normal form: a contradiction. Hence R is not well-defined.

As regarding Examples 4.1, the membership-conditional system is well-defined since the condition $x \in T(F')$ in the third rule is obviously decidable. From the following lemma, the system in Example 4.2 is also well-defined. (The well-defindness in the examples in Section 5 can also be shown in a similar way.)

Lemma 4.1. Let R be a membership-conditional system in which each condition has the form $x \in NF$ (where x may be any variable). Then, R is well-defined.

Proof. Consider the claim: $M \in NF$ (i.e., the irreducibility for M) is decidable for any term M. It is clear that the lemma follows from this claim. We will prove the claim by induction on the size $|M|$ of the term M (i.e., the number of the symbols occurring in M). The case $|M| = 1$ is trivial since M is a variable or a constant. Assume the lemma for $|M| < k$. Then, we must show the lemma for the case $|M| = k$. It is decidable whether M has a redex as a proper subterm, say P, by $|P| < |M|$ and the induction hypothesis. We will show that it is also decidable whether M is a redex. Consider a rule $M_l \triangleright M_r$ *if* $x, \cdots, z \in NF$. Then, M is a redex for this rule iff $M \equiv M_l\theta$ and $x\theta, \cdots, z\theta \in NF$ for some θ. By $|x\theta|, \cdots, |z\theta| < |M|$ and the induction hypothesis, we can decide whether M is a redex for the rule. Thus, testing every rule in R, we can decide whether M is a redex. Therefore, the decidability of $M \in NF$ follows. $\square$

In this paper, we are interested in only well-defined membership-conditional systems. Thus, from here on "a membership-conditional system R" means implicitly that R is well-defined.

Remark. In the membership-conditional system R with undecidable conditions, the rewriting of any term is in general an undecidable problem. Nevertheless, this does not necessarily mean that R is not well-defined; for example, we might indirectly compute the normal forms by using other ways than rewriting.

Remark. A conditional rule $M_l \triangleright M_r$ *if* $P(x)$ [2], where $P(x)$ is some predicate of the variable x, can be translated into a membership-conditional rule $M_l \triangleright M_r$ *if* $x \in T$ where $T = \{N|\ P(N)\}$. Conversely, taking $P(x) \equiv x \in T$, we can also translate a membership-conditional rule $M_l \triangleright M_r$ *if* $x \in T$ into a conditional rule $M_l \triangleright M_r$ *if* $P(x)$. Thus conditional rules of the form

$$M_l \triangleright M_r \ \textit{if}\ P'(x) \wedge P''(y) \wedge \cdots \wedge P'''(z)$$

are essentially equal to membership-conditional rules of the form

$$M_l \triangleright M_r \ \textit{if}\ x \in T', y \in T'' \cdots, z \in T'''.$$

Hence a membership-conditional term rewriting system can be regarded as a conditional term rewriting system in which every condition $P(x, y, \cdots, z)$ can be translated into a condition $P'(x) \wedge P''(y) \wedge \cdots \wedge P'''(z)$ with separated variables.

5. Confluence of Membership Rewriting

This section gives the sufficient conditions for the confluence property of membership-conditional term rewriting systems.

5.1. Normalized Membership Conditions

Let $R = \langle T, \rightarrow \rangle$ be a membership-conditional term rewriting systems and let T' be a subset of the term set T. We say that T' is closed iff

$\forall M \in T' \; \forall N \in T[M \to N \Rightarrow N \in T']$. We say that T' is terminating iff every $M \in T'$ has no infinite reduction $M \to\to\to \cdots$.

For a term set T' closed and terminating, we can define the normalized term set $T'_{nf} = \{M\downarrow \mid M \in T'\}$ where $M\downarrow$ denotes any normal form obtained from M. Note that from the closure and termination properties of T', T'_{nf} is definable and $T'_{nf} \subseteq T'$. Then, the normalized membership-conditional system R_{nf} is defined by replacing each rewriting rule
$M_l \triangleright M_r \; \textit{if} \; x \in T', \cdots, z \in T''$ in R with
$M_l \triangleright M_r \; \textit{if} \; x \in \psi(T'), \cdots, z \in \psi(T'')$. Here, $\psi(T') = T'_{nf}$ if T' is closed and terminating; otherwise $\psi(T') = T'$. $\underset{nf}{\longrightarrow}$ denotes the reduction relation of R_{nf}. From the closed property, it is trivial that $\underset{nf}{\longrightarrow} \subseteq \to$. We will show that if R_{nf} is confluent then R is so.

Lemma 5.1. Let T' be closed and terminating in R and let $M \in T'$. Then, $M \overset{*}{\underset{nf}{\to}} M\downarrow$ for some $M\downarrow \in T'_{nf}$.

Proof. Since T' is terminating, R can reduce M into some $M\downarrow$ by rewriting only innermost $\to$ redex occurrences (i.e., innermost reduction strategy). From the definition of R_{nf}, every innermost $\to$ redex occurrence is an innermost $\underset{nf}{\to}$ redex occurrence. Thus, by tracing the innermost reduction $M \overset{*}{\to} M\downarrow$ by R_{nf}, $M \overset{*}{\underset{nf}{\to}} M\downarrow$ follows. □

Lemma 5.2. We have the diagram in Figure 3.

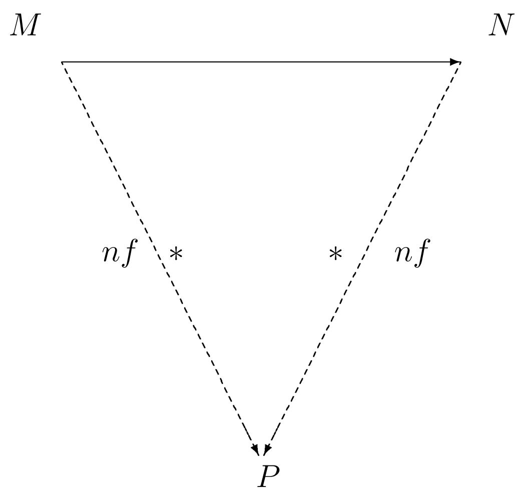

Figure 3. Diagram for Lemma 5.2.

Proof. For example, let R have a reduction $M \equiv C[f(A, A, B)] \to N \equiv C[g(A, B, B)]$ by a rule $f(x, x, y) \triangleright g(x, y, y) \; \textit{if} \; x \in T', y \in T''$, and let T' be closed and terminating and T'' be not (i.e. $\psi(T') = T'_{nf}$ and $\psi(T'') = T''$). Then, the normalized rule is $f(x, x, y) \triangleright g(x, y, y) \; \textit{if} \; x \in T'_{nf}, y \in T''$. From Lemma 5.1, $A \overset{*}{\underset{nf}{\to}} A\downarrow$ for some $A\downarrow \in T'_{nf}$. Hence, R_{nf} have the reductions
$C[f(A, A, B)] \overset{*}{\underset{nf}{\to}} C[f(A\downarrow, A\downarrow, B)]$ and
$C[g(A, B, B)] \overset{*}{\underset{nf}{\to}} C[g(A\downarrow, B, B)]$. By using the normalized rule, $C[f(A\downarrow, A\downarrow, B)] \underset{nf}{\to} C[g(A\downarrow, B, B)]$. Thus, take $P \equiv C[g(A\downarrow, B, B)]$. It is clear that for any M, N, we can always take some P in the same way as for the above example. □

Lemma 5.3. If R_{nf} is confluent then we have the diagram in Figure 4.

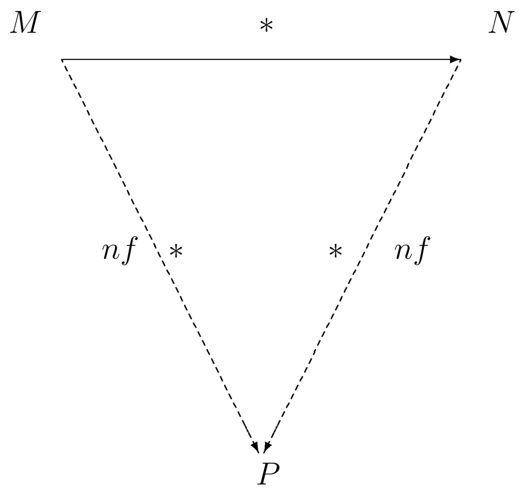

Figure 4. Diagram for Lemma 5.3.

Proof. Let $M \overset{n}{\to} N$, where $\overset{n}{\to}$ denotes a reduction of n ($n \geq 0$) steps. Then we prove the lemma by induction on n. The case $n = 0$ is trivial. Assume the claim for $n-1$ ($n > 0$). Let $M \to M' \overset{n-1}{\to} N$. Then, the diagram in Figure 5 can be obtained, where diagram (1) are shown by Lemma 5.2, diagram (2) by the induction hypothesis, and diagram (3) by the confluence of R_{nf}. □

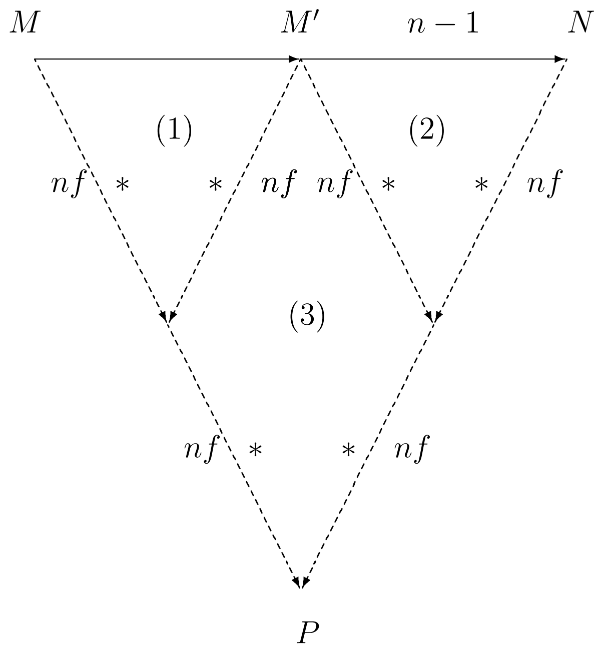

Figure 5. Diagram for the proof of Lemma 5.3.

Theorem 5.1. If R_{nf} is confluent then R is so.

Proof. The diagram in Figure 6 can be obtained, proving diagram(1) by Lemma 5.3, diagram(2) by the confluence of R_{nf}. From $\underset{nf}{\to} \subseteq \to$, the confluence of R follows. □

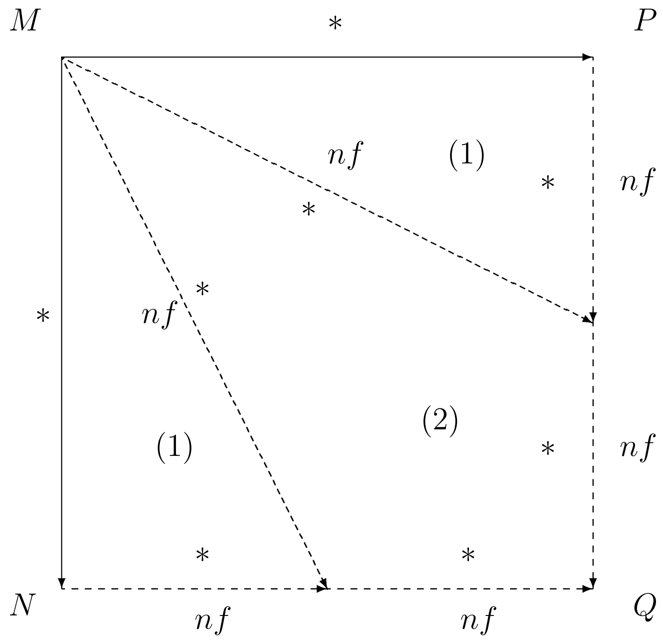

Figure 6. Diagram for the proof of Theorem 5.1.

5.2. Restricted Nonlinear Systems

It is well known that if a term rewriting system is terminating, the confluence can be easily proven by the critical pair lemma [3, 4, 9]. However, if a term rewriting system is nonterminating, it is difficult to prove the confluence of the system. In particular, a system that is nonterminating and nonlinear gives few results to prove the confluence [5, 11].

In this subsection, we study the confluence of membership-conditional term rewriting systems without assuming the terminating property or the linearity. Our key idea to prove the confluence comes from the observation that with appropriate membership conditions, nonlinear systems behave like left-linear systems.

Definition. A restricted nonlinear rule is a membership-conditional rewriting rule in which the nonlinear variables on the left side of the rule must have membership conditions. For the other variables, membership conditions are optional. We say that R is restricted nonlinear iff every rule in R is restricted nonlinear.

For example, the restricted nonlinear rule $f(x, x, y) \triangleright g(x, y, y)$ *if* $x \in T'$ has nonlinear variable x on the left side $f(x, x, y)$. Hence, variable x must have the membership condition $x \in T'$. However, variable y on the left side is linear, thus, membership condition for y is not necessary.

A classical left-linear term-rewriting system is obviously a restricted nonlinear system, because the left-linear system has only linear variables on the left side of the rewriting rules. Thus, the restricted nonlinear system is a natural extension of the classical left-linear system. Indeed, the sufficient criteria for the confluence of restricted nonlinear systems are very similar to that of the classical left-linear systems.

Overlapping between two conditional rewriting rules can be defined in the same way as for two classical rewriting rules except that the substitution must satisfy the membership conditions in the rules. Then, Proposition 3.2 for the confluence of the classical left-linear systems can be extended to the following theorem.

Theorem 5.2. Let a membership-conditional term rewriting system R be nonoverlapping and restricted nonlinear. If every term set T' in the membership conditions is a set of normal forms, i.e., $T' \subseteq NF$, then R is confluent.

Proof. Since nonlinear variables on the left side of the rewriting rules must have normal forms as the substitution's values, the nonlinear variables can be ignored when we treat a sufficient criterion for the confluence. Thus, the confluence of R can be easily proven in the same way as for the classical left-linear and nonoverlapping systems, by tracing the proof in [3, 10] of Proposition 3.2. □

Example 5.1. Consider the membership-conditional term rewriting system R:

$$R \quad \begin{cases} f(x,x) \triangleright 0 \ \textit{if}\ x \in NF \\ f(g(x),x) \triangleright 1 \ \textit{if}\ x \in NF \\ 2 \triangleright g(2) \end{cases}$$

Note that R is nonterminating and nonlinear. Clearly, R satisfies the conditions in Theorem 5.1. Thus, R is confluent. □

In Theorem 5.2, every set T' in the membership conditions must be a set of normal forms. We are now going to relax this restriction on the membership conditions by Theorem 5.1.

Theorem 5.3. Let a membership-conditional term rewriting system R be nonoverlapping and restricted nonlinear. If every term set T' in the membership conditions is closed and terminating, then R is confluent.

Proof. From Theorems 5.1 and 5.2, the theorem follows. □

Example 5.2. Let $F' = \{f, g, 0, 1\}$. Consider the membership conditional term rewriting system R:

$$R \quad \begin{cases} f(x,x) \triangleright 0 \ \textit{if}\ x \in T(F',V) \\ f(g(x),x) \triangleright 1 \ \textit{if}\ x \in T(F',V) \\ 2 \triangleright g(2) \end{cases}$$

It is clear that R is nonoverlapping and restricted nonlinear. Since $T(F',V)$ is closed and terminating, from Theorem 5.3 it follows that R is confluent. □

6. Conclusion

In this paper, we have proposed a new conditional term rewriting system: the membership-conditional term rewriting system. We have shown the sufficient criteria for the confluence of the system under the restricted nonlinear condition.

Many directions for further research come easily to mind. One direction is application to many-sorted systems. Membership-conditional systems can provide a very useful means of constructing hierarchical many-sorted systems.

Application to functional programs is another very interesting direction. Membership-conditional systems can explicitly provide reduction strategy, such as innermost reduction. Hence, using this property, we can offer effective computation for functional programs.

We believe that further research in these directions will exploit the potential of membership-conditional rewriting techniques.

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