An Introduction to Higher-Dimensional Rewriting Theory

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The general methodology of rewriting systems has been applied to various settings: strings, terms, graphs, etc. In this introductory talk, I will present *higher-dimensional rewriting systems* which provide a unifying framework for many of them. These were introduced by Street (as computads) and by Burroni (as polygraphs) with the following motivations. A string rewriting system can be seen as a particular *presentation* of a monoid, describing it by the means of generators and relations. Moreover, when the presentation is confluent and terminating, normal forms provide us with a notion of canonical representative for the elements of the monoid, allowing one to perform many computations on the monoid. The starting point of higher-dimensional rewriting systems is that they should generalize these fruitful tools from monoids to the much richer setting of n-categories.

I will present the nice inductive definition of those rewriting systems, in which an $(n+1)$-dimensional rewriting rule rewrites a rewriting path in an n-dimensional rewriting system, and show how usual rewriting formalisms can be recovered as particular low-dimensional cases. After that, I will explain how usual tools extend to this setting: in particular, I will show that a finite rewriting system can have an infinite number of critical pairs, and present generic ways of constructing termination orders. Finally, we will review some of the applications of those rewriting systems: they can namely be used in order to obtain coherence theorems for various categorical structures (such as MacLane's coherence theorem for monoidal categories), they also found applications in the study of algebraic structures up to homotopy through Koszul duality theory for operads.